Electromagnetic waves

- waves, mean of transporting energy or information
- wave is a function of both space and time
- $\omega \rightarrow$ angular frequency
- $f \rightarrow$ frequency (number of cycles per second)
- $\lambda \rightarrow$ wave length

Center Share

1. Faraday's Law

magnetic form $-V_{emf} = \frac{d\phi}{dt}$ $\phi \rightarrow$ flux through each turn

electric form $-V_{emf} = \oint E \cdot dl = IR$

* Maxwel Said

$$\therefore \oint E \cdot dl = \frac{d\phi}{dt} \rightarrow (1)$$

where $\phi = \oint B \cdot dS$

$$\frac{d}{dt}(\phi) = \frac{d}{dt} \oint B \cdot dS \rightarrow (2)$$

From (1) & (2)

$$\oint E \cdot dl = -\frac{d}{dt} \oint B \cdot dS$$

Scanned by CamScanner

→ From stokes's theorem

$$\int_S (\nabla \times \bar{E}) \cdot dS = \oint E \cdot dL$$

$$\therefore \int_S (\nabla \times \bar{E}) \cdot dS = -\frac{d}{dt}\int_S B \cdot dS$$

$$\therefore \nabla \times \bar{E} = \ominus \frac{dB}{dt}$$

From lenz's Law, the induced current I flows to produce a magnetic field That opposes (B).

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$$ 1st eqn of Maxwell's point form.

if B is Constant $\frac{\partial B}{\partial t} = 0$

$\therefore \nabla \times \bar{E} = 0$

So we study <u>Time varying field.</u>

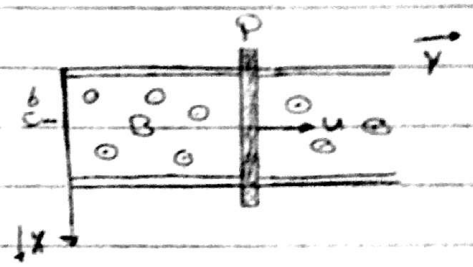
this eqn Show That a time changing magnetic field produce an electric field.

Center Share

Scanned by CamScanner

Example :

$B = 4 \cos 10^6 t \; a_z \; wb/m^2$



$y = 8$ cm

Find V_{emf} ?

$$V_{emf} = -\int \frac{\partial B}{\partial t} \cdot dS$$

Center Share

$$\frac{\partial B}{\partial t} = -4 * 10^6 \sin 10^6 t$$

$$\therefore V_{emf} = \int 4 * 10^6 \sin 10^6 t \cdot dx \cdot dy$$

$$= \int_{x=0}^{6cm} \int_{y=0}^{8cm} 4 * 10^6 \sin 10^6 t \; dx \; dy$$

$$= .06 * .08 * 4 * 10^6 \sin 10^6 t$$

$$= 19200 \sin 10^6 t \quad \text{volt}$$

Scanned by CamScanner

Ampere's Law

$$I_{enc} = \oint H \cdot dL$$

and $\nabla \times \bar{H} = J$

$$\therefore \nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 .$$

non equal zero

But $\nabla \cdot J = \dfrac{\partial \rho_v}{\partial t}$

Center Share

Displacement Current

$$\nabla \times \bar{H} = J + G$$

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot J + \nabla \cdot G$$

$$\underset{0}{\underset{\downarrow}{=}} = \nabla \cdot J + \nabla \cdot G$$

$$0 = -\frac{\partial \rho_v}{\partial t} + \nabla \cdot G$$

$$\therefore \nabla \cdot G = \frac{\partial \rho_v}{\partial t}$$

where $\rho_v = \nabla \cdot D$

$$\therefore \nabla \cdot G = \frac{\partial}{\partial t} \nabla \cdot D = \nabla \cdot \frac{\partial D}{\partial t}$$

$$\therefore G = \frac{\partial D}{\partial t}$$

Scanned by CamScanner

$$\nabla \times \bar{H} = J + \frac{\partial D}{\partial t}$$

$$= J_c + J_d$$

J_c : Conduction Current density ($J_c = \sigma E$) (الكثافة الحالية التوصيلية) توصيل

J_d : Displacement Current density ($J_d = \frac{\partial D}{\partial t}$) $D = \varepsilon E$

* Maxwell's equation in point form

① $\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$

② $\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \frac{\partial \varepsilon \bar{E}}{\partial t}$

③ $\nabla \cdot D = \rho_v$

④ $\nabla \cdot B = 0$

Center Share

* Maxwell's equation in integral form

① $\oint \bar{E} \cdot d\bar{L} = -\int_s \frac{\partial B}{\partial t} \cdot dS$

② $\oint \bar{H} \cdot d\bar{L} = I + \int_s \frac{\partial D}{\partial t} \cdot dS$

③ $\oiint_s D \cdot dS = \int_{vol} \rho_v \, dv = Q_{enc}$

④ $\oiint_s B \cdot dS = 0$

Scanned by CamScanner

موجة بتنتشر في موصل

• Wave propagate in a general medium (lossy medium)

(E & H has only one component { E_x or E_y or E_z } , { H_x or H_y or H_z })

assume

$$\bar{E} = \bar{E}_x \, \hat{a}_x$$

$$\bar{H} = \bar{H}_y \, \hat{a}_y$$

Center Share

من

• From maxwell's equations

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\begin{vmatrix} \overset{+}{\hat{a}_x} & \overset{-}{\hat{a}_y} & \overset{+}{\hat{a}_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \bar{E}_x & \bar{E}_y \to 0 & \bar{E}_z \to 0 \end{vmatrix} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{E}_x \left(\frac{\partial}{\partial z} \hat{a}_y - \frac{\partial}{\partial y} \hat{a}_z \right) = -\frac{\partial \bar{B}}{\partial t} \qquad B = \mu \bar{H} \, \hat{a}_y$$

no comp. at direct z for $\bar{H}$

$$\boxed{\frac{\partial \bar{E}_x}{\partial z} \hat{a}_y = -\mu \frac{\partial H}{\partial t} \longrightarrow (1)}$$

Scanned by CamScanner

. from equation ① E must have Component of (z)

$$\frac{\partial E}{\partial z} = \checkmark$$

if E has no z comp.

$$\frac{\partial E}{\partial z} = \text{Zero}.$$

From the Maxwell's equation

Center Share

② $\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d$

$$\begin{vmatrix} \overset{+}{a_x} & \overset{-}{a_y} & \overset{+}{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cancel{\bar{H}_x} & \bar{H}_y & \cancel{\bar{H}_z} \end{vmatrix} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$$

$$-H_y \left(\frac{\partial}{\partial z} \hat{a}_y - \frac{\partial}{\partial x} \hat{a}_z \right) = \sigma E_x + \varepsilon \frac{\partial E_x}{\partial t}$$

$$-\frac{\partial H_y}{\partial z} \hat{a}_x + \cancel{\frac{\partial H_y}{\partial x}} \hat{a}_z = \sigma E_x + \varepsilon \frac{\partial E_x}{\partial t}$$

$$\therefore \frac{\partial \bar{H}_y}{\partial z} \hat{a}_x = -E_x \left(\sigma + \varepsilon \frac{\partial}{\partial t} \right)$$

②

. also H must have z Component

Scanned by CamScanner

* هذا ما يعني ال Plane wave

① عندي مركبة ال E في اتجاه واحد فقط ، i.e. x ،

② و " " " H في اتجاه عمودي ، i.e y ،

③ هي في اتجاه الانتشار ال power بتاعتهم
عمودي عليهم وفي اتجاه ال (Z)

الأن نفرض في الحصول على الشكل — نمثل

* now we aim to get form represent E propagation
and H propagation

From ①

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial}{\partial t} (\nabla \times H)$$

$$\cancel{\nabla (\nabla \cdot E)}^{\,0} - \nabla^2 E = -\mu \frac{\partial}{\partial t} \left(\sigma E + \varepsilon \frac{\partial E}{\partial t} \right)$$

$$-\nabla^2 E = -\mu \frac{\partial}{\partial t} \left(\sigma E + \varepsilon \frac{\partial E}{\partial t} \right)$$

$$\frac{\partial}{\partial t} = j\omega$$

$$\nabla^2 E = \mu (j\omega) (\sigma E + \varepsilon j\omega E)$$

$$= j\omega\mu\sigma E - \omega^2 \mu \varepsilon E$$

Center Share

Scanned by CamScanner

$$\nabla^2 E = (j\omega\mu\sigma - \omega^2\mu\varepsilon)\, E$$

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$E_x = E_x^+ e^{-\gamma z} + E_x^- e^{\gamma z}$$

$$\therefore \bar{E} = E^+ e^{-\gamma z}\, \hat{a}_E + E^- e^{\gamma z}\, \hat{a}_E$$

also

$$\bar{H} = H^+ e^{-\gamma z}\, \hat{a}_H + H^- e^{\gamma z}\, \hat{a}_H$$

$$\because \gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{(j\omega\mu\sigma - \omega^2\mu\varepsilon)}$$

Center Share

$$\gamma = (j\omega\mu\sigma - \omega^2\mu\varepsilon)^{1/2}$$

propagation constant

$$\gamma = \alpha + j\beta$$

α: attenuation constant (تخفيف)

β: phase constant, wave number

- Calculate (α & β)

$$\gamma^2 = \alpha^2 - \beta^2 + 2j\alpha\beta$$

Scanned by CamScanner

$$\therefore \mathrm{Re}(\gamma^2) = -\omega^2 \mu \varepsilon$$

$$\therefore \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon$$

and

$$|\gamma^2| = \alpha^2 + \beta^2 = \sqrt{(\omega\mu\sigma)^2 + (\omega^2 \mu\varepsilon)^2}$$

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

$$\therefore 2\alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} - \omega^2\mu\varepsilon$$

$$2\alpha^2 = \omega\left(\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} - \omega\mu\varepsilon\right)$$

$$2\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} - 1\right]} \rightarrow (1)$$

$$\therefore \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1} + 1\right]} \rightarrow (2)$$

$$\gamma = \alpha + j\beta$$

$$|\gamma| = \sqrt{\alpha^2 + \beta^2}$$

$$|\gamma^2| = \alpha^2 + \beta^2$$

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$$

$$|\gamma^2| = \sqrt{(\omega^2\mu\varepsilon)^2 + (\omega\mu\sigma)^2}$$

Center Share

Scanned by CamScanner

Equations for propagation constant at different media

مثالي ضياع عازل

① lossless dielectric (perfect dielectric)

(free space ε_0, μ_0)

غير موصل

$\sigma = 0$ (no conductivity)

• العوازل الخانقة هي أفضل المواد الحديثة لنقل الموجات

$$\gamma^2 = j\omega\mu\cancel{\sigma}_{0} - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{-\omega^2\mu\varepsilon} = j\omega\sqrt{\mu\varepsilon}$$

Center Share

$$\therefore \gamma = \alpha + j\beta$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} \quad \text{rad/m}$$

موصل مثالي

② perfect Conductor

$$\sigma = \infty$$

$$\therefore \alpha = \infty \qquad \text{attenuation} = \infty$$

The wave has sharp attenuation at the surface of the perfect conductor material (no field)

Scanned by CamScanner

عازل الفقد

② lossy dielectric

$\sigma \neq 0 \qquad \sigma \ll \omega\varepsilon \qquad \left|\frac{J_c}{J_d}\right| \ll 1$

$\therefore \frac{\sigma}{\omega\varepsilon} \ll 1$

$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$

$\gamma = \sqrt{j\omega\mu}\sqrt{\sigma + j\omega\varepsilon} \qquad j = -\frac{1}{j}$

$= \sqrt{j\omega\mu} \cdot \sqrt{j\omega\varepsilon} \cdot \sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}$

$= j\omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)^{\frac{1}{2}} \qquad (1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} + \dots$

$= j\omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma}{2j\omega\varepsilon}\right)$

$= j\omega\sqrt{\mu\varepsilon} + \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$

Center Share

$\therefore \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad Np/m$

$\therefore \beta = \omega\sqrt{\mu\varepsilon} \quad rad/m$

Scanned by CamScanner

④ Good Conductor

$$\sigma \gg \omega\varepsilon \qquad \left|\frac{J_c}{J_d}\right| \gg 1$$

$$\frac{\omega\varepsilon}{\sigma} \ll 1$$

$$\therefore \gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$$

Center Share

$$\gamma = \sqrt{j\omega\mu\sigma\left[1 - \frac{\omega^2\mu\varepsilon}{j\omega\mu\sigma}\right]}$$

$$= \sqrt{j\omega\mu\sigma}\left[1 + j\frac{\omega\varepsilon}{\sigma}\right]^{1/2}$$

$$= \sqrt{j\omega\mu\sigma}\left[1 + \frac{j\omega\varepsilon}{2\sigma}\right] \qquad \sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$= \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(\sqrt{\omega\mu\sigma}\right)\left(1 + \frac{j\omega\varepsilon}{2\sigma}\right)$$

$$= \left(\frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}} + j\frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}}\right)\left(1 + j\frac{\omega\varepsilon}{2\sigma}\right)$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} - \cancelto{0}{\frac{\omega\varepsilon\sqrt{\omega\mu\sigma}}{2\sqrt{2}\,\sigma}} + j\sqrt{\frac{\omega\mu\sigma}{2}} + j\cancelto{0}{\frac{\omega\varepsilon\sqrt{\omega\mu\sigma}}{2\sqrt{2}\,\sigma}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{Np/m}$$

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{rad/m}$$

Scanned by CamScanner

Wave impedance (η)

$$\bar{E}_x = E^+ e^{-\gamma z} + E^- e^{\gamma z} \hat{a}_x$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -j\omega \mu H$$

Center Share

only for forward wave (E & H)

$$\cancel{-}\gamma E^+ \cancel{e^{-\gamma z}} = \cancel{-} j\omega\mu H \rightarrow (H^+ \cancel{e^{-\gamma z}})$$

$$\gamma E = j\omega\mu H$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$$

$$= \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

when $R = \frac{V}{I}$

$$\therefore \eta = \frac{E}{H} = \frac{j\omega\mu}{\gamma}$$

$$\therefore \eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

Scanned by CamScanner

where η is Complex

$$\eta = |\eta| e^{j\theta_\eta}$$

Center Share

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}.$$

$\tan\theta_t = \tan 2\theta_\eta$ → loss tangent

$\theta = 2\theta_\eta$ → loss angle

$$\therefore E(z) = E_o^+ e^{-\gamma z} + E_o^- e^{\gamma z}$$

$$E(z,t) = \mathcal{R}_e\left(E(z)\, e^{j\omega t}\right)$$

as $E(z,t)$ is an Explicit function

$$= \mathcal{R}_e\left(E_o^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right)$$

$$= E_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_E$$

$$\therefore H = \frac{E_o^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)$$

$$H = \frac{E_o^-}{|\eta|} e^{\alpha z} \cos(\omega t + \beta z - \theta_\eta)$$

Scanned by CamScanner

Special cases of wave impedance

① perfect dielectric

$\sigma = \text{zero}$

$$\eta = \sqrt{\frac{j\omega\mu}{\cancel{\sigma} + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$

Center Share

② free space

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\pi \ \Omega$$

③ lossy medium (dielectric)

$$\frac{\sigma}{j\omega\varepsilon} \ll 1$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} \cdot \frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \left(1 + \frac{\sigma}{j\omega\varepsilon}\right)^{-1/2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} + j\frac{\sigma}{2\omega\varepsilon}\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} + j\frac{\sigma}{2\omega\varepsilon}\sqrt{\frac{\mu}{\varepsilon}}$$

Scanned by CamScanner

4 lossy medium (Good Conductor)

$$\frac{\omega\varepsilon}{\sigma} \ll 1$$

$$\gamma = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} \cdot \frac{1}{\sqrt{1 + j\frac{\omega\varepsilon}{\sigma}}}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma}} \left(1 + j\frac{\omega\varepsilon}{\sigma}\right)^{-1/2}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma}} \left(1 - \cancel{\frac{j\omega\varepsilon}{2\sigma}}\right) \qquad \sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

Center Share

5) perfect Conductor

$$\sigma = \infty$$

$$\gamma = \text{Zero}$$

Scanned by CamScanner

c in depth (δ)

. The Signal is meant to be exist and to be detected until it reaches e^{-1} from its Max

, So e^{-1} is equally to $e^{-\alpha z}$ ($e^{-1} = 36\%$)

$\therefore -1 = -\alpha z$

$$\boxed{z = \delta = \frac{1}{\alpha}}$$

① perfect dielectric

Center Share

$\alpha = 0$, $\delta = \infty$

. The wave not suffer from any attenuation so it can travel ∞ length through the perfect dielectric (free space)

② lossy dielectric

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Scanned by CamScanner

3) Good Conductor

$$\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

② & ③ Shows that the skin depth (δ) increase as the (σ) decrease

④ perfect Conductor

$$\sigma = \alpha = \infty$$

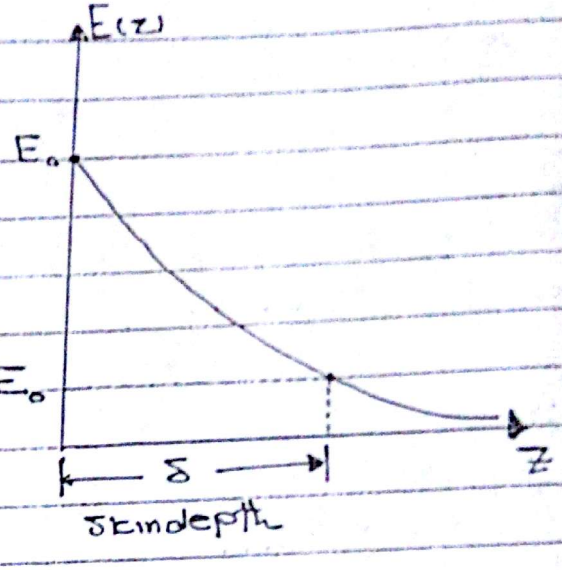
$$\delta = \text{Zero}$$

the Wave Cann't penetrate the perfect Conductor as it is reflected

Center Share

Scanned by CamScanner

- it is clear as $\sigma \neq 0$ the wave is attenuated while it travels through the medium.



- After the signal reaches 3.6% from its initial value it'll be very attenuated and not detectable

- The distance after which the signal attenuated is called the skin depth (δ)

Center Share

Scanned by CamScanner

②

Example :

lossy dielectric has impedance $200\angle 30^\circ\ \Omega$
and Magnetic field $H = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2} x)\ a_y$ A/m

find E and α , skin depth ? , wave polarization

Sol.

The wave travels at $a_E = \hat{a}_x$
$a_H = \hat{a}_y$

$\therefore \hat{a}_k = \hat{a}_E \times \hat{a}_H$

$\therefore a_E = -\hat{a}_z$

$H = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2} x)\ \hat{a}_y$

$H_o = 10$

$|\eta| = \frac{E_o}{H_o}$

$\therefore E_o = |\eta| H_o = 10 * 200$

$\therefore E = 2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \overset{\theta_\eta}{\frac{\pi}{6}})\ (-a_z)$

$= -2000\, e^{-\alpha x} \cos(\omega t - \frac{x}{2} + \frac{\pi}{6})\ \hat{a}_z$ V/m

Scanned by CamScanner

$$\therefore \beta = \frac{1}{2}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$\therefore \tan(\underbrace{2\theta_{\eta}}_{=30^\circ}) = \frac{\sigma}{\omega\varepsilon}$$

$$\therefore \frac{\sigma}{\omega\varepsilon} = \sqrt{3}$$

$$\therefore \frac{\alpha}{\beta} = \frac{\left(\sqrt{(1+3)}-1\right)^{1/2}}{\left(\sqrt{1+3}+1\right)^{1/2}} = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{1}{2\sqrt{3}} \quad Np/m$$

$$\therefore \delta = \frac{1}{\alpha} = 2\sqrt{3} \quad m$$

- Wave polarization → is the direction of the electric field

So, the wave is polarized along the (z) direction.

Scanned by CamScanner

Example :

In a lossless medium for whic $\eta = 60\pi$, $\mu_r = 1$

$H = -.1 \cos(\omega t - z)\,\hat{a}_x + .5 \sin(\omega t - z)\,\hat{a}_y$ A/m

Find E , ω , ε_r ??

→ For lossless medium

$\sigma = 0 \qquad \alpha = 0$

$\because \eta = 60\pi$

$\therefore \eta = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi$

$\frac{\sqrt{\mu_0 \mu_r}}{\sqrt{\varepsilon_0 \varepsilon_r}} = 60\pi$

$\therefore \varepsilon_r = 4$

$\because H = -.1 \cos(\omega t - z)\,\hat{a}_x + .5 \sin(\omega t - z)\,\hat{a}_y$

$\therefore \beta = 1$

$\beta = \omega\sqrt{\mu\varepsilon} = 1 \qquad \therefore \omega = \frac{1}{\sqrt{\mu\varepsilon}}$

$\therefore \omega = 1.5 * 10^8$ rad/sec

Scanned by CamScanner

The wave propagates at z

$$a_k = \hat{a}_z$$

$$a_H = -\hat{a}_x \qquad a_H = \hat{a}_y$$

$$a_E = \hat{a}_y \qquad a_E = \hat{a}_x$$

$$\therefore \eta = \frac{E_{o_1}}{H_{o_1}}$$

$$\therefore E_{o_1} = \eta H_{o_1} \qquad E_{o_2} = \eta H_{o_2}$$

$$\therefore \overline{E} = 60\pi * .1 \cos(\omega t - z)\, a_y$$

$$+ 60\pi * .5 \sin(\omega t - z)\, a_x =$$

$$= 6\pi \cos(1.5 * 10^8 t - z)\, \hat{a}_y + 30\pi \sin(\omega t - z)\, \hat{a}_x \ \text{V/m}$$

Scanned by CamScanner

Example :

A uniform plane wave propagating in a medium

$$E = 2 e^{-\alpha z} \sin(10^8 t - \beta z) a_y \ V/m$$

If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$, $\sigma = 3$.

Find (α ; β ; H)

Sol.

we have to determine the type of medium → lossy dielectric / → good conductor

determine the loss tangent

$$\frac{\sigma}{\omega \varepsilon} = \frac{3}{10^8 * \varepsilon_0} = 3393 \gg 1$$

The medium is a good conductor

$$\therefore \alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$= \left(\frac{10^8 * 4\pi * 10^7 * 20 * 3}{2} \right)^{1/2}$$

$$= 61.4$$

$\alpha = 61.4$ Np/m

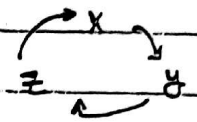
$\beta = 61.4$ rad/m

Scanned by CamScanner

$\therefore E = 2 e^{-61.4z} \sin(10^8 t - 61.4z)\, \hat{a}_y$

$\hat{a}_k = \hat{a}_z$

$a_E = \hat{a}_y$



$\therefore a_H = -\hat{a}_x$

$H_o = \frac{E_o}{|\eta|}$

$\therefore |\eta| = \sqrt{(\text{real})^2 + (\text{img})^2}$

$= \sqrt{\left(\sqrt{\frac{\omega\mu}{2\sigma}}\right)^2 + \left(\sqrt{\frac{\omega\mu}{2\sigma}}\right)^2} = \sqrt{\frac{\omega\mu}{\sigma}}$

$= \sqrt{\frac{800\pi}{3}}$

$\therefore \theta_\eta = \tan^{-1}\left(\frac{\text{img}}{\text{real}}\right) = 45^\circ = \frac{\pi}{4}$

$\tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon}$

$\therefore \theta_\eta = 45^\circ = \pi/4$

$\therefore H = \frac{2}{\sqrt{\frac{800\pi}{3}}} e^{-61.4z} \sin(10^8 t - 61.4z - \pi/4)\,(-\hat{a}_x)$

$= -69.1 \times 10^{-3} e^{-61.4z} \sin(10^8 t - 61.4z - \frac{\pi}{4})\, \hat{a}_x$

A/m

Scanned by CamScanner

* power and the poynting vector

$$\nabla \times E = \mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \qquad * E$$

$$E \cdot (\nabla \times H) = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t} \rightarrow (1)$$

$$\nabla \cdot (\underset{\downarrow H}{A} \times \underset{\downarrow E}{B}) = \underset{\downarrow E}{B} (\nabla \times \underset{\downarrow H}{A}) - \underset{\downarrow H}{A} \cdot (\nabla \times \underset{\downarrow E}{B})$$

$$\nabla \cdot (H \times E) + H (\nabla \times E) = E \cdot (\nabla \times H)$$

from (1)

$$\nabla \cdot (H \times E) + H \cdot (\nabla \times E) = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t}$$

$$\nabla \cdot (H \times E) - \mu \frac{\partial H^2}{\partial t} = \sigma E^2 + \varepsilon \frac{\partial E^2}{\partial t}$$

$$-\nabla \cdot (E \times H) = \sigma E^2 + \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} + \frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

$$\oint_V \nabla \cdot (E \times H)\, dV = -\oint_V \sigma E^2 \cdot dV - \frac{\partial}{\partial t} \oint_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right) dV$$

from divergence theorem

$$\oint_S (E \times H) \cdot dS = -\oint_V \sigma E^2 \cdot dV - \frac{\partial}{\partial t} \oint_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right) dV$$

Total power

$\therefore$ poynting vector $\mathcal{P} = E \times H$ watt/m^2

Scanned by CamScanner

* Poynting vector ($\mathcal{P}$)

$$\mathcal{P}(z,t) = E \times H$$

* instantaneous poynting vector

$$\mathcal{P}_{ave}(z) = \frac{1}{T}\int_0^T \mathcal{P}(z,t)\,dt$$

$$= \frac{1}{2} \mathcal{R}_e (E \times H^*)$$

$$= \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta \; \hat{a}_z$$

* The total time average power

$$P_{ave} = \oint_S \mathcal{P}_{ave}(z) \cdot dS \quad \text{Watt}$$

Scanned by CamScanner

Example :

In a nonmagnetic medium

$$E = 4 \sin(2\pi * 10^7 t - .8x)\ \hat{a}_z \ \text{V/m}$$

Find

① ε_r , η

② The time average power

③ the total power crossing $100\,cm^2$ of plane $2x + y = 5$

Sol.

nonmagnetic means lossless $\alpha = 0$ $\sigma = 0$ $\mu_r = 1$

① $\beta = \omega\sqrt{\mu\varepsilon} = .8$

$$.8 = 2\pi * 10^7 \sqrt{\mu_0 * \varepsilon_0 \varepsilon_r}$$

$$\varepsilon_r = 14.59$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 10\pi^2 = 98.7\ \Omega$$

② $\mathcal{P} = E \times H = \frac{E_0^2}{|\eta|} \sin^2(\omega t - \beta x)\ \hat{a}_x$

$$\mathcal{P}_{ave} = \frac{1}{T}\int \mathcal{P}\, dt = \frac{E_0^2}{2\eta}\, a_x$$

$$= \frac{(4)^2}{2 \times 10\pi^2} = .081\ \hat{a}_x \ \text{W/m}^2$$

Scanned by CamScanner

$$P_{ave} = \int_S \mathcal{P}_{ave} \cdot dS = \mathcal{P}_{ave} \cdot S \cdot \hat{a}_n$$

$$\therefore P_{ave} = (.81 * 10^{-3}\, \hat{a}_x) \cdot (100 * 10^{-4}) \cdot \hat{a}_n$$

$$a_n = \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{2^2 + 1^2}} = \frac{2a_x + a_y}{\sqrt{5}}$$

$$P_{ave} = .081\, a_x * 100 * 10^{-4} * \frac{2a_x + a_y}{\sqrt{5}}$$

$$\hat{a}_x \cdot \hat{a}_x = 1$$

$$\hat{a}_x \cdot \hat{a}_y = \text{Zero}$$

$$P_{ave} = 8.1 * 10^{-4} * \frac{2}{\sqrt{5}}$$

$$= 724.5\ \mu W$$

Scanned by CamScanner